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SEMICLASSICAL DECAY OF EXCITED STRING STATES ON LEADING REGGE TRAJECTORIES

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Abstract

We study the decay of hadrons based on a semiclassical string model. By including quark mass effects we find that the width to mass ratio Γ/m is an increasing function of m, which increases most rapidly for massive quarks. This is consistent with the available data. The decay probability of hadrons on the leading Regge trajectories is computed taking the effect of the string rotation into account. The resulting decay probability is no longer uniform along the length of the string but varies in a manner that is in qualitative agreement with the available data. We argue in favour of possible experiments that would test our predictions more accurately and help open a window to the nonperturbative aspects of QCD.

1. Introduction

Much of the physics of confinement can be understood on the basis of a simple string picture of hadrons. In particular, a linear potential between a quark and an antiquark and a linearly rising Regge trajectory are simple, immediate consequences of this picture. The linearity of Regge trajectories is, in fact, the most striking evidence that we have for the string model. The spin **6** meson of mass 2450 which lies on the (still very) linear $\rho - A_2$ trajectory is dramatic evidence for the existence of a string out to distances of almost three fermis! Each newly discovered high spin meson on this trajectory extends our knowledge of strings by about 1/4 fermi. The reproduction of linear potentials was one of the earliest and most encouraging successes of lattice gauge theories. The string model has successfully been applied to the hadronization of quarks produced in $e^+ \, e^-$ collisions. The string model is useful, fecund and well established.

Study of the hadronic string model gave rise to superstring theory where truly one dimensional strings are (The) fundamental objects. It is clear that QCD strings are not one dimensional but have finite extent like flux tubes and thus cannot be treated by fundamental theory. Nevertheless they are still predominantly stringy and therefore we may learn about them from studying superstring theory. Recently, there has been much interest in studying the relationship between fundamental string models and the three dimensional flux tubes connecting physical quarks.[1]

The success of the string picture and its physical evocativity behooves us to explore how far we can extend the simple, semi-classical picture of hadrons. Since the existence of linearly rising Regge trajectories is at once the main test and main manifestation of the string picture it pays to see what further physics can be extracted from this simple physical system. The essence of the string model for hadrons is the relationship between the energy (mass) and the angular momentum of a spinning string (or quasi one dimensional object

such as a flux tube). The energy and the angular momentum are given by

$$E = \frac{\pi k}{\omega}, \qquad J = \frac{\pi}{2}\,\frac{k}{\omega^2}, \tag{1.1}$$

where ω is the angular frequency of the string and k is the string tension. Finite mass quarks can be placed at the ends with interesting implications.[2–4]

The string model becomes semi-classical when we account for its breaking. In the presence of a large chromo electric field a $q - \overline{q}$ can tunnel out of the vacuum and eat up some of the string.[5] This process has been studied since the earliest days of QED and, at least for electrodynamics, is well understood.[6] The string will break with equal probability per unit four volume. The string breaking probability for QED is

$$p = \frac{\alpha E^2}{2\pi^2} \sum_n \frac{1}{n^2} \exp \left(\frac{-n\pi m^2}{|eE|} \right), \tag{1.2}$$

where m is the quark mass, $k = eE$ is the string tension expressed in terms of the electric field E. Heavier quarks are very much less likely to pop out of the vacuum. This string breaking picture has been successfully employed in fragmentation models which provide qualitative support for the ansatze.[7]

For a string state the total width is proportional to the total decay probability. In the semiclassical string breaking model this probability is directly proportional to the string length. On the leading Regge trajectory, since the mass is proportional to the length, the picture predicts the ratio of width to mass, Γ/m, should be constant. For mesons in the particle data book the width to mass ratio ranges from .05 to .2, providing at best only weak qualitative agreement. As we shall see some of the trends inherent in this data can be explained by the more detailed picture we develop below.

In Section II we consider the corrections to the simplest string model by allowing for massive quarks at the ends of the strings. For massive quarks a state of *given* energy will have less of its energy in the string than if the quarks were massless. Thus the length of string, and hence its decay probability will be smaller than it would be for

the massless case. Asymptotically as the hadron mass increases (and the quark masses become negligible with respect to the string energy) Γ/m will increase to its constant universal value. Nevertheless for light mass quarks (m <200-300 MeV) the trajectories will remain nearly linear. The massive quark takes up almost exactly the energy *and* angular momentum of the string it replaces maintaining an almost linear relationship between J and E.

The leading Regge trajectory comes about because the string is spinning. This will introduce centrifugal effects which will change the simple minded prediction. In Section III we model the centrifugal effects and find that strings prefer to decay symmetrically to states of angular momentum $\sim J/2$ rather than decaying by cascading down to a state with nearby J. These centrifugal effects also cause the decay probability for particles on the Regge trajectory to be different than that used in the string fragmentation models.

It strikes as unfortunate that because of the rush to higher energy physics the field of "low energy" Regge spectroscopy has languished with almost no progress in the last ten years. Yet it appears that this is the most productive place to experimentally explore the details of the confinement mechanism of QCD. Surely this is a worthy topic for extended effort even as we test the (basically perturbative) predictions of the Standard Model to ever higher precision. There are many important conceptual questions about the nature of confinement that experimental data can elucidate. For instance, in fundamental string theory there is evidence that the width of the particles on the leading trajectory will level off with increasing mass (and J) whereas the naive physical flux tube model predicts a linear increase.[8] Experimental studies providing more accurate widths and much higher spin states would distinguish between these predictions and could thus provide clues as to the relationship between fundamental strings and semi-classical, thick strings. In general any detailed information about the interactions and breakings of clearly delineated, extended strings must be useful. We urge our experimental colleagues to revive this once

active field of particle physics research.

2. Classical Corrections: Finite Mass Quarks

When we place massive quarks at the ends of the string the simple relationship Eq. (1.1) between mass and angular momentum is modified and becomes a parametric relationship :

$$M = \frac{m_1}{(1-v_1^2)^{\frac{1}{2}}} \; + \; \frac{m_2}{(1-v_2^2)^{\frac{1}{2}}} + \frac{k}{\omega} \int_{-v_2}^{v_1} \frac{dv}{(1-v^2)^{\frac{1}{2}}}, \tag{2.1}$$

$$J = \frac{m_1 v_1^2}{\omega(1-v_1^2)^{\frac{1}{2}}} \; + \; \frac{m_2 v_2^2}{\omega(1-v_2^2)^{\frac{1}{2}}} + \frac{k}{\omega^2} \int_{-v_2}^{v_1} dv \frac{v^2}{(1-v^2)^{\frac{1}{2}}}, \tag{2.2}$$

$$v_i = \left(1 + \left(\frac{m_i \omega}{2k}\right)^2\right)^{\frac{1}{2}} - \frac{m_i \omega}{2k}, \tag{2.3}$$

where v_i are the velocities of the quarks of masses m_i.

Intuition suggests that the string will shrink and may slow its rotational velocity compared to the case of massless quarks at the ends. To verify this we consider the case when the velocity of the massive quark is relativistic. This occurs when $\frac{m\omega}{2k}$ is small. By using Eq. 2.1-2.3 we find the leading corrections of order $\left(\frac{m\omega}{2k}\right)$ cancel out implying that the relationship between J, M, and are very similar to the massless case :

$$E = \frac{k}{\omega}\left(\pi + \frac{4\sqrt{2}}{3}\left(\frac{m\omega}{2k}\right)^{\frac{3}{2}}\right),$$

$$J = \frac{k}{\omega^2}\left(\frac{\pi}{2} - \frac{4\sqrt{2}}{3}\left(\frac{m\omega}{2k}\right)^{\frac{3}{2}}\right). \tag{2.4}$$

Neglecting terms of order $\left(\frac{m\omega}{2k}\right)^{\frac{3}{2}}$ these are formally identical to the massless case. To order $\frac{m\omega}{k}$, for fixed J (or E), ω and E (or J) are unchanged. The velocity of the endpoint, and hence the string length $v = \omega L$ decreases by $\frac{m\omega}{2k}$. The relationship Eq. 1.1 is now

$$J = \frac{E^2}{2\pi k} - \frac{4}{3}\left(\frac{2}{\pi}\right)^{\frac{1}{4}}\left(\frac{m}{\sqrt{k}}\right)^{\frac{3}{2}} J^{\frac{1}{4}}. \tag{2.5}$$

The correction to linearity is both relatively small and slowly varying with J. It can easily be masked when including the Regge intercept (or quantum defect) $J = 1/(2\pi k)E^2 + \alpha_0$ in a fit to the Regge trajectory. [3−4]

When finite mass quarks are at the ends, the length of the string, to which the semi-classical width is proportional, increases faster than the energy with increasing E. The string grows "faster" than in the massless case. For small mass, and hence $\frac{m\omega}{2k}$ small, we find (keeping only terms of order $\frac{m\omega}{2k}$) that

$$\frac{L}{E} = \frac{2}{k\pi} - \left[\frac{(m_1 + m_2)}{2kE}\right]. \tag{2.6}$$

This changes the naive prediction that the ratio of the width to particle mass should be a universal constant for particles on leading Regge Trajectories. To compare to experiment we must chose reasonable values for the masses.

In a previous fit to the Regge trajectories of the strange and the nonstrange mesons the values $m(u,d) = 175$ and $m(s) = 345$ were selected [4]. We use these values to compute the string length for the corresponding mesons. In Fig. 1 we display the ratio of the length to the energy of the K^* trajectory as calculated from Eq. 2.1 - 2.3 with $\alpha' = .92$. We have multiplied the ratio L/E by 1/29 so as to normalize this ratio to the observed ratio of the $K^*(3)$ width to its mass. We remind the reader that this curve is not a fit but a prediction (up to normalization) from the parameters of Ref. 4. Fig. 2 is the corresponding plot for the $\rho - A_2$ trajectory. As the experimental points we plot the average value of Γ/m for the $I = 1$ and the $I = 0$ particles. The string model does not distinguish between different isospin states and so this is the natural object to compare to. Experimentally the small pion mass and G parity play a large and possibly distorting role in the decay patterns for the low mass states and the averaging is a reasonable way

to eliminate the wide variations. Had we not done so the data are patternless and reflect the mass and quantum number constraints. The dramatic success of the K^* trajectory is vitiated by the poor correspondence in the $\rho - A_2$ case. Nevertheless, even for this case, there is evidence for a gradual increase. (We have used the same normalization as for the K^* trajectory as would be required by the string model.) The relatively sharp increase of the "predicted" curve is driven by the mass of the light quark. The data can be reproduced by a smaller mass of about 50-100 MeV for $m(u, d)$. The point we want to emphasize here is that the meson data show the width increases faster that the mass, markedly so for the strange mesons and much less so for the $\rho - A2$ trajectory. The string model with finite quark mass, qualitatively, predicts just this trend. We can also apply the model to high spin baryons on the leading baryon Regge trajectories. Here too linearity of the Regge trajectory implies a string model. One of the three quarks is excited producing a string like structure connecting it with the two remaining quarks (diquark[9]). By appropriate choice of diquark mass, $m = .975$, the string with massive quarks at the end readily accommodates the trend in Γ/m. See Fig. 3.

3. Quantum Corrections: Centrifugal Barriers to Tunnelling

A simple model for string breaking involves quark pair creation by the strong chromo-electric field inside the string. Although a field theory treatment is required for a correct calculation the essential physics is contained in a semiclassical analysis of tunneling, from the vacuum, of a quark antiquark pair. Since we are looking at semi-quantitative results this semiclassical picture will be sufficient. Creation of massive $q - \overline{q}$ from the vacuum in a relativistic effect and so we employ the Klein-Gordon equation in the WKB amplitude for tunneling. In this treatment we are ignoring the spin of the quarks so that the Klein Gordon equation should be sufficient. A quark of mass m pops out of the vacuum ma-

terializing after vaporizing some of the string and leaving behind an antiquark hole. On its way out it experiences a kr linear potential. The WKB expression for the tunneling event is

$$P \sim \exp\left(-2\int_0^{r_c}\sqrt{(E-V(r))^2-m^2}\,dr\right) = \exp\left(-\frac{\pi m^2}{k}\right). \tag{3.1}$$

r_c is the classical turning point, and $E=-m$. Remarkably this is identical to the exact field theory result of Euler, Heisenberg, and Schwinger.[5]

We are interested in how this simple, one-dimensional, result changes when, as is appropriate for particles on Regge trajectories, the string is rapidly rotating. The quark will now have to overcome a centrifugal barrier to tunnel out. This can be accounted for by using WKB for the radial Klein Gordon equation.

$$P \sim \exp\left(-2\int_0^{r_c}\sqrt{(E-V(r))^2-m^2-\frac{l(l+1)}{r^2}}\right). \tag{3.2}$$

The l appearing here is the angular momentum of the string picked up by the quark as it tunnels out. Strictly speaking we should replace $l\,(l+1)$ by $(l+\frac{1}{2})^2$ incorporating the Langer correction.[10] This correction emphasis the region near $r=0$ (as we show below $l\to 0$ as $r\to 0$) which is where our ignorance of the physics is greatest and will force us to introduce an extra parameter as a cutoff. Since we do not expect our results to be quantitative we chose to continue with the uncorrected centrifugal term rather than add another, uncontrolled parameter. This should still be a reliable qualitative guide to the centrifugal effects. We thus need to know the angular momentum carried by the vaporized string.

Consider a point a distance R away from the pivot point B (center of mass) of a string rotating with angular frequency ω (see Eq. 2.1-2.3). Due to the string's rotation this point moves with a velocity $v=\omega R$ perpendicular to the string. Consider a quark tunneling from R to a point $R+r$ away from the pivot point (see Fig. 4., where B is the center of mass of the whole string AE before breaking and CD is the portion that vaporizes.) The

quark acquires the angular momentum of the vaporized string. This angular momentum, relative to the point R is

$$l(r) = \int_R^{R+r} \frac{k d\rho v(\rho)\rho}{\sqrt{1-v(\rho)^2}}, \tag{3.3}$$

where $v(\rho)$ is the relative velocity of the string element with respect to R :

$$v(\rho) = \frac{\omega\rho}{1-\omega^2(\rho+R)R} \ . \tag{3.4}$$

In many situations of physical interest the value of r is small and a small r expansion is adequate. This yields

$$l(r) = \frac{k\omega}{(1-\omega^2R^2)}\left[\frac{r^3}{3} + \frac{R\omega^2}{(1-\omega^2R^2)}\frac{r^4}{4} + \quad ...\right]. \tag{3.5}$$

When this approximation is valid we can neglect the quadratic term l^2 in the tunneling amplitude and find for the string breaking probability for a spinning string

$$P \sim \exp -\frac{m^2\pi}{k}\left\{\frac{\left(1+\frac{w}{6m(1-\omega^2R^2)}\right)^2}{\left(1-\frac{w^3R}{4k(1-\omega^2R^2)^2}\right)^{3/2}}\right\}. \tag{3.6}$$

Compared to the one dimensional case there is an additional factor in the curly brackets in the exponent of the decay probability. This extra suppression is position dependent, as we might suspect for a spinning string. Tunneling is more difficult near the end of the string than at the fulcrum. For instances for a high angular momentum string with ω=.05 and m=.1 (k=1/5.8) the probability is 30% higher for the string to break at the center (R=0) than near the end (R=18). This has several interesting phenomenological consequences in the semiclassical string model. First we see that the string is more likely to break near its middle in a symmetric way into two high angular momentum states rather than splitting near the end of the string and cascading down by the emission of a "pion" to a high spin precursor. The semiclassical string model quantifies this simple physics conclusion by providing a specific recipe for the centrifugal suppression. This type of suppression is consistent with the sparse data that exists.

The semiclassical string model has seen its widest usage as an input for the so-called Lund Fragmentations model. In that situation there is no need to take into account the centrifugal effects because we believe the process is one dimensional to a high degree of accuracy. An input to this process is the decay probability per unit length of the string. This value is usually assumed to be the same as that deduced from the widths of particles on the leading Regge trajectories. However because of the centrifugal effects calculated above these two quantities are not the same. Neither the string fragmentation model nor the Regge string decay model is so precise that this is significant but it is good to keep in mind when comparing ever sophisticated versions of the two semiclassical string models.

As the quarks tunnel out from the vacuum they eat up some of the string. This restricts the amount of string available for decay. For instance (ignoring the centrifugal effects) the quarks require a minimum length of $2m/k$ in order to tunnel out. Similarly the string cannot break within a distance of $2m/k$ of the ends. This further modifies the simple relation between width and length of the string as only a part of the string is available for decay. Including the centrifugal effects will require that even more string must vaporize in order for the quarks to materialize. The length of the vaporized string depends on its position on the string via the R dependence appearing in Eq. (3.3). Since more string is required for larger R this effect will further suppress non symmetrical decays with respect to the symmetrical decays. The R dependence in Eq. (3.3-3.6) is not dramatic however and so there will be only mild position dependence.

Phenomenologically the simplest approach to these effects is to set the Γ/m ratio equal to $L - \Delta$ ignore the R dependence and consider Δ as a parameter. This provides the model with sufficient robustness to accommodate the data incorporated in Fig. 1-3. We return to this data and relax the requirement that the masses are given by the best fit to the Regge trajectory. We still restrict the quark masses to the range found acceptable [4] for reproducing Regge trajectories. Using 75 MeV for the light quarks, 400 MeV for

the strange quark, 1000 MeV for the Λ diquarks and a value of .2 GeV^{-1} for Δ we can easily represent the decay data. This is displayed in Figures 5-8. The quark masses are reasonable but somewhat smaller than usual constituent quark masses while the Λ diquark mass is on the high side. The value of Δ is smaller than we might have expected from the above physical discussion based on semiclassical tunneling. The nulceon data also show the expected trend in Γ/m but the uncertainties in Γ for high J states is so large that a graph of Γ/m vs. L/E is not illuminating. It is quite clear that the simple, but physical, semiclassical string model provides a good description of the total decay widths of particles on the leading meson and baryon Regge trajectories.

The relevance of the semiclassical picture is satisfyingly supported by the numerical comparison of the phenomenologically extracted string decay probability with the most naively calculated value. Eq. (1.2) is the semiclassical decay probability per unit four volume. To convert this to the probability of string decay we must multiply by the cross sectional area of the string, a factor of two for the quark spin and a factor of at least two for the two (or three?) flavors of light quarks that can independently pop out of the vacuum. Taking a flux tube radius of $\sim$1/300 MeV and using $k = 1/5.8$, $m = .075$ the semiclassical result 1/65 is comparable in magnitude to the 1/27.5 used in Figures 5-8. Given the myriad of uncertainties in the model and the values of the parameters (especially the flux tube radius) this confluence is most encouraging.

We have not displayed our model for the charm meson trajectory because of its embarrassing inadequacy. The D^{**} meson, while sitting appropriately on a charm meson trajectory [4], is exceedingly narrow, $\Gamma \sim 25$ MeV. This is somewhat troublesome for any meson model, certainly our own. In the simplest picture we expect heavy quark mesons to be narrower than their light quark counterparts because the string, for the same J, is shorter for the heavier meson (by a factor of $\sqrt{2}$ in the infinite mass heavy quark limit). The tunneling effects discussed above and embodied in the parameter Δ emphasize this

decrease. For instance, there is no "symmetric" decay possible for a heavy quark meson. Still the width does seem inordinately small. Nevertheless the width of the first excited state on a heavy meson trajectory is very sensitive to these effects and we can easily imagine that a more accurate treatment of them will produce the observed width. As higher excitations are discovered we expect the widths to approach the typical widths of the other mesons.

4. Conclusions

The string model provides a beautiful, physical picture of quark confinement. It strongly motivates the linear part of the quark anti quark potential which has met with great success in heavy quark systems. Strings predict and account for the remarkable linearity of both meson and baryon Regge trajectories. In this paper we went beyond the spectroscopic aspects of the string model and explored the implications of string breaking for hadron decays. The mechanism for breaking is envisioned as the tunneling from the vacuum of $q-\overline{q}$ pair. This leads to the prediction that the ratio of the hadron width to its mass is a universal constant, the string breaking probability. If we take isospin averages, (in order to average over phase space effects due to the pion mass) the data are in rough agreement with the expectation. A definite deviation is, however, evident. The width to mass ratio is an increasing function of the particle mass. This trend is, in fact, exactly what the most immediate refinements to the naive model predict. Quarks have mass and when placed at the ends of strings they shorten the string length for constant mass hadron. Thus Γ/m while still proportional to L/m is now a rising function of m which goes to a universal constant asymptotically. The fact that tunneling requires a finite length of string reinforces this trend. Indeed the data are consistent with the combination of these two effects and thus supportive of semiclassical string picture of decay.

Our major results are

1) To first order in quark mass, massive quarks shorten the length of the string but do not alter the linearity of the Regge trajectory. This explains the dramatic success of the massless quark string model for Regge trajectories.

2) When we isospin average the meson widths, the width to mass data occupy a narrow range from .055 o .15 and show a definite trend of increasing with the mass of the hadron.

3) Straightforward incorporation of finite quark mass effects and a semiclassical treatment of decay reproduce this trend in the data.

4) The "experimental" value of the string decay constant is very broadly consistent with the theoretical tunneling probability.

5) The centrifugal barrier to tunneling gives rise to a preference for the string to decay symmetrically. This is consistent with an experimental preference for such decays and with decays of highly excited fundamental string in 26 dimension but, possibly, not consistent with fundamental string decays in less than the critical dimension.

6) Because of the positional dependence of the centrifugal barrier strictly speaking the string decay consistent is different for states on the Regge trajectory from that for use in fragmentation functions.

Our results demonstrate the continued vigor of the string model and its robustness in confronting and illuminating the dynamical process of hadron decay. We hope it provides encouragement for future studies, both experimental and theoretical, of hadronic strings. The problem of understanding quark confinement as a consequence of the QCD Lagrangian is a formidable one that has so far resisted twenty years of vigorous attack. We are however, possessed of a physically compelling model that has an excellent chance of emerging from the final solution to the confinement problem. Let us fully exploit it.

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Figure Captions

Fig. 1 The isospin averaged Γ/m for particles on the $\rho - A_2$ trajectory plotted on the same plot as L/E for the corresponding string model states. The experimental Γ/m has been multiplied by 29.

Fig. 2 Same as Fig. 1 for K^* trajectory.

Fig. 3 Same as Fig. 1 for Λ trajectory.

Fig. 4 String after tunneling of $q\overline{q}$ and consequent vaporization of a string segment.

Fig. 5 String model, as explained in text, for $\rho - A_2$ trajectory. Γ/m is multiplied by 27.5.

Fig. 6 Same as Fig. 5 but for K^* trajectory.

Fig. 7 Same as Fig. 5 but for ϕ trajectory.

Fig. 8 Same as Fig. 5 but for Λ trajectory.